ABSTRACT
We investigate the utility of PKP coda waves for studying weak scattering from small-scale heterogeneity in the mid-mantle. Coda waves are potentially a useful probe of heterogeneity at shallow depths because this energy is not preferentially scattered near the CMB, as PKP precursors are, but is due to scattering at all depths. PKP coda waves have not been used for this purpose historically because of interference with other late-arriving energy due to near-surface resonance and scattering. We have analyzed 3624 recordings of PKP precursors and coda made by stations in the IRIS Global Seismographic Network. To study the range and time dependence of the scattered waves, we binned and stacked envelopes of the recordings. PKP scattered waves increase in amplitude rapidly with range as predicted by scattering theory. At ranges below ~125°, we predict and observe essentially no scattered energy preceding PKP. Coda amplitudes at these ranges are independent of range and provide an estimate of energy due to near-surface effects we can expect at all ranges. We use the average coda amplitude at ranges from 120 to 125° to correct coda amplitudes at other ranges. PKP coda waves show a strong dependence on time and range and are clearly influenced by scattering in the lower mantle. PKP coda waves, however, do not provide a tighter constraint on the vertical distribution of mantle heterogeneity than is provided by precursors. This is due, in part, to relatively large scatter in coda amplitudes as revealed by a resampling analysis. Modeling shows that PKP coda amplitudes are not highly sensitive to the vertical distribution of heterogeneity in the mantle. To illustrate this we consider single-scattering in two extreme models of mantle heterogeneity. One allows heterogeneity just at the CMB; the other includes heterogeneity throughout the mantle. The amplitudes of precursors are tightly constrained by our stack and support our earlier conclusion that small-scale heterogeneity is uniformly distributed throughout the lower mantle. The best-fit model includes 8 km scale length heterogeneity with an RMS velocity contrast throughout the mantle of 1%.

INTRODUCTION
Since the pioneering work of Haddon and Cleary in the early 1970’s, the high-frequency energy that precedes the inner core phase PKP(df) has been known to result from scattering in the mantle (Haddon, 1972; Haddon and Cleary, 1974; Figure 1). These scattered waves arrive free of interference from late-arriving near-surface scattered waves and thus provide a unique window into the small-scale structure of the deep Earth. Numerous studies have used recordings of these precursors to investigate the physical properties of small-scale heterogeneity in the mantle (e.g. Doornbos 1972; Husebye et al., 1976; Hedlin et al., 1997). Although a recent analysis by Hedlin et al. (1997) used GSN recordings of the precursors to study heterogeneity throughout the lower mantle, the geometry of the precursors favors studies of the deepest mantle as the preponderance of early arrivals come from great depth. Hedlin et al. (1997) modeled the weak scattering that gives rise to precursors using the Born approximation and found that the gradual growth of the precursors with time and range favored a model that included heterogeneity distributed with equal strength at all depths in the lower mantle. Models that include heterogeneity just at the CMB or within D” produce early arrivals, and thus sharper onsets than are seen in the data. This surprising result awaits confirmation by an independent analysis, as models involving multiple scattering near the CMB might also be able to explain the PKP precursor observations.

PKP coda waves are due largely to near surface scattering and resonance, but also include contributions from scattering throughout the mantle. Evidence that lower mantle scattering contributes energy to PKP coda can be found in raw data (Figure 2). Recorded PKP coda amplitude variations with time and
range mirror those observed in the precursors. This basic observation suggests that we might be able to put tighter constraints on the strength on heterogeneity in the mantle using coda waves provided that we are able to correctly compensate coda amplitudes for near surface effects. It is this idea that we test in this paper.

**DATA**

We have analyzed 3624 recordings of PKP precursors and coda that were made between day 151 of 1988 to the end of 1999 by stations in the Incorporated Research Institutions for Seismology (IRIS) Global Seismographic Network (GSN). These recordings were selected from a much larger dataset on the basis of pre-event noise levels. We rejected traces that exhibited noise levels that were high or varied sharply with time. The events in the culled dataset occurred at depths from the near-surface to 657 km. Sixty percent of the events occurred within 100 km of the free-surface. The events were assigned body wave magnitudes between 5.0 and 7.3. Although the GSN stations are distributed across the globe, the tight range limits on the recordings (120 to 145°) resulted in a rather uneven sampling of the Earth (Hedlin and Shearer, 2000). Some areas are not sampled at all.

**PRELIMINARY DATA ANALYSIS**

The traces in Figure 2 reveal a strong dependence of the precursors and coda waves on range and time. The precursors and coda increase in amplitude relative to PKP from 120 to 144°. Energetic coda is seen in most traces to 60 seconds after the arrival of PKP. The traces, however, also reveal variations that are clearly not global in origin but are regional. One notable example of this is the trace at ~ 137° which is dominated by PKP. This trace implies weak scattering within the small volume of the mantle it samples. An earlier paper (Hedlin and Shearer, 2000) examined regional variations in scattering. This paper is concerned with global characteristics of the data. To constrain global average scattering strength, we binned each recording into the appropriate 1° bin, after correcting for event depth. Each trace was filtered between 0.7 to 2.5 Hz and converted to an envelope. Prior to stacking, we applied a small, downward, adjustment of the amplitude to remove pre-event noise. The binning procedure grouped recordings simply on the basis of recording range.

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**Figure 1.** Ray paths and travel times of the different branches of PKP for a source at the surface. The retrograde ab and prograde bc branches refract through the outer core. The cd branch is reflected from the inner core boundary. The df branch refracts through the inner core. Scattering involving the ab or bc branch can produce precursory arrivals as well as later arrivals that contribute to the df coda. Scattering of df will not cause df precursors but will also contribute to the df coda. Two hypothetical scatterers (labeled CMB and LM) produce precursors that can arrive no earlier than the labeled curves in the lower panel. The deepest scatterers will give rise to the earliest precursors. Although we have depicted scattering from PKP to P near the receiver, precursors can also result from scattering from P to PKP on the source side.
and not on the location of either the source or the receiver. The stacked traces thus provide an image of the average characteristics of the arrivals.

The stacks shown in Figure 3 indicate that coda amplitudes at ranges from 120 to 125° are nearly independent of range. No precursor energy is observed at these ranges. Between 125 and 144° we observe a gradual increase in the amplitude of the precursors and coda. The precursors at these ranges increase in amplitude from the onset to PKP. After PKP, the coda amplitudes decay gradually with time. Scattering simulations (Hedlin et al., 1997) indicate that most of the energy at ranges below 125° is due to near surface effects. To illustrate the amount by which the amplitude of the precursors and coda increases from the low levels seen at ranges below 145° we plotted a copy of the average stack from 120 to 125° at all ranges (see the dashed traces). This range dependence is unlikely due to near surface effects but we believe arises from scattering within the mantle. At 145°, the traces are dominated by PKP as the b caustic arrives at this range. All scattered waves are small in amplitude relative to PKP. The suites of curves in Figure 3 indicate the minimum travel time for single-scattered energy scattered at depths ranging from the CMB to 2200 km above. This figure illustrates that most scattered waves will follow PKP.

THE DEPTH EXTENT OF MANTLE SCATTERING

Hedlin et al. (1997) considered the factors listed in the previous section and modeled a stack of 1600 PKP precursor recordings using Rayleigh-Born scattering theory (Chernov, 1960) applied to the exponential autocorrelation function. The modeling indicated that the global average small-scale heterogeneity has a scale length of ~8 km and is distributed uniformly throughout the lower-mantle with an RMS velocity contrast of 1%.

We are attempting to re-examine the vertical distribution of heterogeneity in the mantle using coda waves to check the earlier study. This requires careful consideration of the paths energy might take through the core before or after scattering in the mantle. Propagation through the mantle from the surface to the CMB involves no triplications. Propagation from the CMB back to the surface after propagation through the core is along one of four branches of PKP (Figure 1). The earliest arriving branch at ranges between ~119 and 132° is bc which represents prograde refraction through the outer core. The ab branch lags behind by a few seconds at most. The ab and bc branches alone can yield precursors to PKP. These branches were considered by Hedlin et al. (1997). The other branches can be
associated only with coda arrivals. The \( df \) branch arrives first at ranges from 109 to \( \sim 119^\circ \) and beyond 132° and thus makes possible scattered coda arrivals across a greater range interval than is possible from either the \( ab \) or \( bc \) branches. Precursors and coda to \( PKP \) are due to scattering from a small volume of the mantle near the source or the receiver.

The object of this paper is to determine if \( PKP \) coda will allow us to place a tight constraint on the relative importance of scattering at shallow and great depths in the mantle. To investigate the vertical distribution of heterogeneity in the mantle we divide the mantle into thin (20 km) layers, from the CMB to the crust, and calculate the response of each individual layer to the incident wavefield. The statistics of the heterogeneity are invariant within each layer and are allowed to change only in strength between layers. The calculation used in this paper is identical to the single-scattering theory used by Hedlin et al. (1997) except we predict the amplitudes of \( PKP \) coda as well as precursors. As a result, we also consider \( df \) refracted scattered waves. As observed by Hedlin et al. (1997), the earliest and largest contributions will come from the deepest layers.

By summing the energy of different combinations of these kernels we can explore the effect of scattering within different mantle depth intervals. To model the data using these kernels it is necessary to take into account the \( ab \), \( bc \) and \( df \) branches of \( PKP \) as well as late-arriving energy that results from scattering and resonance in the near-surface. We have observed that the coda amplitudes are independent of range when the recording is made at ranges below about 125°.

The stacking procedure combines traces from widely separated regions of the Earth. The averaging of coda amplitudes removes regional variations and produces an estimate of the average near-surface energy levels we can expect at all ranges. The range independent average also includes energy that results from scattering in the mantle and propagation through the inner core along the \( df \) branch. Our simulations indicate that the \( df \) branch has a weak dependence on range when compared with the \( ab \) and \( bc \) branches which are strongly influenced by the \( b \) caustic. The simulations also clearly show that regardless of what scattering model is used, the \( ab \) and \( bc \) branches produce essentially no scattered energy before or after \( PKP \) at ranges below 125°.

To take into account, and remove from consideration, near-surface effects and the inner core refracted scattered energy, we subtract the average coda amplitude observed at ranges below 125° from all bins. The average stacked coda from 120 to 125° possesses minor variations with time. To ensure that this structure is not introduced into the stacks by the correction procedure, we approximate the average stack using a standard coda decay
We assume the coda envelope amplitude as a function of time, $A(t)$, can be described as follows:

$$\log[A(t)] = \log[A_0] - \alpha \log(t - t_p),$$

where $t_p$ is the time of the PKP onset, $A_0$ is the onset amplitude, $\alpha$ is a free fitting parameter. We deduct the best fit exponential decay function from the bins at all ranges. The correction applies only to the data points at times after the onset of PKP.

The result is given in Figure 4. We have shown the bins between 130 and 141°. The bins at greater ranges are difficult to interpret due to proximity to the $b$ caustic. The bins at ranges below 130° provide little insight as there is very little energy before PKP or after PKP once the correction for $df$ and the near-surface effects is made. At each range we show the stacked, and corrected, data and the standard errors from a resampling analysis (Efron and Tibshirani, 1991). The standard errors increase after the arrival of PKP. This is expected as coda amplitudes are largely due to near surface effects. These effects depend on the depth of the event. Most events were located within 100 km of the surface. Some events were > 600 km in depth. While we have observed robust range independent estimates of coda amplitudes at ranges below 125°, and have inferred similar behavior at greater ranges, wide variability in the amplitudes that are present in individual traces is revealed by the resampling analysis.

Despite the scatter in the corrected PKP coda amplitudes shown in Figure 4, it is clear that the coda energy remains well above zero even after corrections for near-surface scattering are applied. This implies that scattering from the CMB and/or deep mantle is a significant contributor to PKP coda energy at these ranges. This result could have been anticipated from Figure 3, given the observed increase in PKP coda amplitude for those ranges where such deep scattering could occur. One goal of this paper is to test whether these PKP coda observations can place new constraints on the depth extent of lower mantle scattering.

Predicted results from two different models are shown in Figure 4. One model (represented by the dashed curves) allows 8-km scale length heterogeneity within a 20-km-thick mantle layer immediately
above the CMB and models the effect of scattering at or very close to the CMB (short-wavelength topography on the CMB would produce similar effects). The second model (solid curves) allows 8-km heterogeneity to be uniformly distributed throughout the mantle. For each model, we adjusted the RMS amplitude of the velocity variations to obtain the best fit to the \( PKP \) precursors. Our simulations reveal that coda decay rates are nearly independent of the vertical distribution of heterogeneity in the mantle. The CMB-only model predicts slightly more rapid decay of coda amplitudes after \( PKP \). This difference is seen at all ranges. Taking into account uncertainty about the coda amplitudes as determined from a resampling analysis, our stacks indicate that both models are in nearly equal agreement with the data. Both predict an increase in energy before \( PKP \) and then a gradual decay.

However, as noted in Hedlin et al. (1997), the two models do not produce equal fits to the \( PKP \) precursors. The model that includes heterogeneity just at the CMB produces onsets that are more rapid than observed. In addition, the CMB scattering model requires much larger velocity perturbations to achieve the same amplitude of scattering. The whole mantle model requires RMS velocity perturbations between 1 and 2% whereas the CMB scattering model requires perturbations of 5.5 to 7.5% within the deepest 20 km of the mantle.

**CONCLUSIONS**

Our analysis of 3624 teleseismic recordings of \( PKP \) precursors and coda indicates that coda arrivals do not provide a useful constraint on the vertical distribution of heterogeneity within the mantle. There are two reasons for this. While the onset of scattered energy depends strongly on the vertical distribution of heterogeneity, the coda decay rates are nearly insensitive to this. The second reason is that the statistical variability of coda amplitudes is high. The standard errors of the stacked data are greater than the difference between the coda amplitudes predicted by the two models.

Our analysis of simulated and recorded data indicates that \( PKP \) precursors remain the most effective probe of small-scale heterogeneity in the mantle. Specifically, the increase in the amplitude of the precursors with time and increasing range appears to be a diagnostic indicator of the vertical distribution of the heterogeneity. Our analysis of 3624 high-quality teleseismic recordings confirms the conclusions of Hedlin et al. (1997). \( PKP \) precursors are consistent with scattering uniformly distributed throughout the lower mantle and not with scattering confined to the CMB. The preferred model requires ~ 1% RMS heterogeneity with a scale length of 8 km at all depths.

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