On the electric current in the D$''$ layer

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Abstract. The electric current in the D$''$layer is due to leakage electric current from the core and induction by time variation of the magnetic field in the region. Toroidal magnetic field becomes zero at the CMB if the mantle is a perfect insulator. However, it can be non-zero with actual mantle due to the leakage electric current. The electric current and magnetic field in the layer produce Lorentz force responsible for electromagnetic core-mantle coupling. To detect the electric current in the D$''$layer is essential to pose observational constraints on the geodynamo and core-mantle coupling. Observational effort to detect the electric current has been made using long baseline submarine cables. Also, theoretical estimate of the electric current in the D$''$layer has been conducted using a simple kinematic dynamo model under the effect of the electrical conductivity of the mantle. The latest view of the observatinal and theoretical results are presented in the paper.

1 Introduction

Electric field measurements using very long submarine cables have been done as a part of the electromagnetic component of the Ocean Hemisphere Network Project. The submarine cable voltages contain a lot of geophysical information such as motional induction due to the flow in the ocean, electrical conductivity of the crust and mantle, and the electric current originated in very deep interior of the Earth. Detecting the electric current from deep interior, which is generated by the geodynamo, is one of the main purposes of the observation. Segregating this signal from the others can be done understanding the time-scale differences among the processes. Typical time scale of electric field variation of deep interior origin is 10 years or longer, which is much longer than the time-scales possesed by the signals originated from the other processes.

Observing the electric field from deep interior is equivalent to measuring the electric current in the D$''$layer, which is believed to have conductivity much larger than the other part of the mantle (Poirier (1993), Poirier et al. (1996), Cohen et al. (1997)). The electric current in the layer is generated by two sources; one is the electric current generated by the geodynamo that leaks into the mantle, and the other is the induced current due to time variation of the magnetic field in the layer. The former is the origin of the toroidal field at the CMB, i.e. it can be used to obtain the strength of the field.

Several attempts to find the strength and distribution of the toroidal magnetic field at the CMB using the submarine cable data had been made. Lanzerotti et al. (1985) obtained approximate DC value of the electric field from the data of 4 to 85 days and concluded that the strength of the toroidal field at the CMB is about the same as that of the poloidal field there. Using the DC value may cause problems on obtaining correct strength of the toroidal field especially if the data length is not sufficiently long. Aliasing likely cause wrong estimate of the DC value of the electric field. Moreover, the measured DC values of the electric field needs careful attention because it could be contaminated by chemical potential between the electrodes and the surrounding material. Shimizu et al. (1998) made a similar attempt but using time dependent part of the electric field. The data of about four years are used for their estimates. They assumed that the electric field variation is due entirely to thirty-year variation of the geomagnetic main field and obtained an approximate upper bound of the toroidal field strength as 1-10 times larger than the poloidal field at the CMB, depending on the mantle conductivity. These estimates should be renewed and improved after more data, long and improved coverage on the Earth, become available.

The electric current in the D$''$layer, if it is detected more accurately, can be used to calculate Lorentz force in the layer. This enables us to estimate the electromagnetic torque on the mantle. As it is well known, the core and mantle are changing their angular momentum in time-scale of longer than about 5 years. The electromagnetic torque is one of the candidates of the cause of the coupling between the core and mantle.

The electromagnetic torque may be calculated from geomagnetic field variations observed at the surface of the Earth (e.g. Holme 1998) using a lot of assumptions. Only the variational part of the torque due to variation of poloidal field and flow at the core surface can be obtained. The part caused by the leakage current from the core cannot be obtained only from the geomagnetic field data. Estimation of the torque due to the leakage current is necessay for more complete discussions of the electromagnetic core-mantle coupling, and the electric field measured using submarine cables has the potential to accomplish it.

To improve the result of the strength of the toroidal field at the CMB and to obtain correct electromagnetic torque from the submarine cable voltages and surface magnetic field data may be possible only after longer data with improved coverage of the Earth's surface are available. Only further measurements using existing network and creating new networks of submarine cables will answer the problem. Nevertheless, it is interesting to compare the electric field we have obtained so far with the electric current in the D″layer simulated using certain models. In this study, electromagnetically consistent electric current in the D″layer is calculated using a kinematic dynamo model. The results can be applied not only to the comparison but also to the design of new submarine cable network

2 Kinematic dynamo problem

Suppose that the radius of the core (i.e. dynamo region) is r_c and that of the Earth is r_a. Also, let the conductivity of the core and mantle be σ (constant) and $\sigma_M(\mathbf{r})$, respectively.

Assume that the α-effect, a parameterized electromotive force generation by small-scale flows and fields, is contributing the magnetic field generation in the core.

The governing equations are, in the core ($0 \leq r \leq r_c$),

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} + \alpha \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{1}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{2}$$

and in the mantle ($r_c < r \leq r_a$),

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times (\eta_M \nabla \times \mathbf{B}), \quad \nabla \cdot \mathbf{B} = 0, \tag{3}$$

where η and η_M are magnetic diffusivities in the core and mantle,

$$\eta = \frac{1}{\mu_0 \sigma}, \quad \eta_M = \frac{1}{\mu_0 \sigma_M}. \tag{4}$$

Continuity of the magnetic field, horizontal electric field and radial electric current are the boundary conditions at the core-mantle boundary and any other boundaries that have conductivity jump in the mantle (including the surface of the Earth.)

In this kinematic problem, flow $\mathbf{v}$ and the α-effect $\alpha(r, \theta, \phi)$ are specified and the variation and distribution of $\mathbf{B}$ is calculated. Usually we assume that the flow satisfies

$$\nabla \cdot \mathbf{v} = 0, \tag{5}$$

i.e. the fluid is incompressible or Boussinesq fluid, and the α-effect is axisymmetric.

Because $\mathbf{B}$ and $\mathbf{v}$ are solenoidal, they can be written with poloidal ($\mathbf{S}$, $\mathbf{s}$) and toroidal ($\mathbf{T}$, $\mathbf{t}$) vectors as

$$\mathbf{B} = \sum_\beta (\mathbf{S}_\beta + \mathbf{T}_\beta), \quad \mathbf{v} = V \sum_\alpha (\mathbf{s}_\alpha + \mathbf{t}_\alpha), \tag{6}$$

where

$$\mathbf{S}_\beta = \nabla \times \nabla \times (S_\beta(r) Y_\beta \hat{\mathbf{r}}), \quad \mathbf{T}_\beta = \nabla \times (T_\beta(r) Y_\beta \hat{\mathbf{r}}), \tag{7}$$

$$\mathbf{s}_\alpha = \nabla \times \nabla \times (s_\alpha(r) Y_\alpha \hat{\mathbf{r}}), \quad \mathbf{t}_\alpha = \nabla \times (t_\alpha(r) Y_\alpha \hat{\mathbf{r}}), \tag{8}$$

Y_γ is spherical harmonics, and $\hat{\mathbf{r}}$ is the unit vector in radial direction. The α effect may be expanded similarly using spherical harmonics, and

$$\alpha = A \sum_\alpha G_\alpha(r) Y_\alpha. \tag{9}$$

Note that V and A, typical amplitudes of flow and α-effect, are taken as the controlling parameters for the kinematic dynamo problem.

By using orthogonality of the spherical harmonics, the equations for the poloidal and toroidal functions in the core may be written as the following (see Bullard and Gellman (1954), Roberts and Stix (1972));

$$\begin{aligned}\frac{\partial S_\gamma}{\partial t} = \eta \nabla_\gamma^2 S_\gamma + V \sum_{\alpha,\beta} [(s_\alpha S_\beta S_\gamma) \\ +(t_\alpha S_\beta S_\gamma) + (s_\alpha T_\beta S_\gamma)] \\ +A \sum_{\alpha,\beta} [(G_\alpha S_\beta S_\gamma) + (G_\alpha T_\beta S_\gamma)]\end{aligned} \tag{10}$$

$$\begin{aligned}\frac{\partial T_\gamma}{\partial t} = \eta \nabla_\gamma^2 T_\gamma + V \sum_{\alpha,\beta} [(s_\alpha S_\beta T_\gamma) \\ +(t_\alpha S_\beta T_\gamma) + (s_\alpha T_\beta T_\gamma) + (t_\alpha T_\beta T_\gamma)] \\ +A \sum_{\alpha,\beta} [(G_\alpha S_\beta T_\gamma) + (G_\alpha T_\beta T_\gamma)],\end{aligned} \tag{11}$$

where

$$\nabla_\gamma^2 = \frac{\partial^2}{\partial r^2} - \frac{n_\gamma(n_\gamma + 1)}{r^2}, \tag{12}$$

and the terms such as $(t_\alpha S_\beta T_\gamma)$ represent interaction between flow and field, and this particular case shows the generation of T_γ (toroidal field) from the interaction of toroidal flow t_α and poloidal field S_β (i.e. this represent the ω-effect.) Equations (10) and (11) can be discretised using finite difference method for numerical calculations.

Same formulation can be done for the equations in the mantle, equation (**??**), by expressing the magnetic diffusivity η_M using spherical harmonic expansion. This enables the calculation of electromagnetic field under the influence of 3D conductivity structure in the mantle. However, we are going to consider a simpler case in this paper supposing that the the electrical conductivity in the mantle is 1D and it depends only on r. Another simplification can be made if the time-scale separation of secular variation and magnetic diffusion time in the mantle is significant. Since the former (decadal) is considered to be much longer than the latter (one year), the electric current in the mantle, in effect, responds immediately to the forceing at the CMB to the first order. Therefore, steady-state solution of the electric current in the mantle can be employed for the problem. For steady state, poloidal magnetic field becomes a potential field in the mantle that is the same as the poloidal field of non-conducting case. On the other hand, the toroidal field is different from non-conductiong case. The equation governing T in the mantle is written as

$$\sigma_M \frac{\partial}{\partial r}\left(\frac{1}{\sigma_M}\frac{\partial T}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial^2 T}{\partial \theta^2} = 0. \quad (13)$$

The solution for this problem is written as

$$T(r) = a_n R_{n1} + b_n R_{n2}, \quad (14)$$

where $R_{n1}(r)$ and $R_{n2}(r)$ are linearly independent functions satisfying equation (**??**). Analitical solutions to this problem can be found for simple distribution of σ_M. If σ_M is constant, the two linear independent solutions are

$$R_{n1}(r) = r^{n+1}, \quad R_{n2} = r^{-n}. \quad (15)$$

See Roberts and Lowes (1961) for solutions for other conductivity distributions.

We suppose that the mantle has layered structure and constant conductivities in each layer. The toroidal function T in the D$''$layer has the form (**??**) with (**??**). This can be used as the boundary condition to the problem in the core. This simplifies the numerical calculations significantly.

Now, the bondary condistions for S and T are needed. These are written as, at the CMB ($r = r_c$) and other boundaries in the mantle,

$$< S >= 0, \quad < T >= 0 \quad (16)$$

$$\left\langle \frac{\partial}{\partial r}\frac{S}{r} \right\rangle = 0, \quad \left\langle \frac{1}{\sigma}\frac{\partial T}{\partial r} \right\rangle = 0, \quad (17)$$

and at the surface of the Earth $r = r_a$,

$$T = 0, \quad \frac{dS_n^m}{dr} + \frac{n}{r}S_n^m = 0, \quad (18)$$

where angle brakets indicate the difference between the two side of the boundaries.

As the first step to this problem, we consider only axisymmetric magnetic field. This can be done because the α-effect is considered in this problem and anti-dynamo theorem is violated by this.

The steady state problem is an eigenvalue value problem to find the condition for self-exciting dynamo for the specified velocity and the α-effect. The eivenvalues, toroidal field at the CMB, and electric current in the D$''$layer are calculated.

Decadal variation of magnetic field may be created using the kinematic dynamo model applying velocity field variation of decadal time-scale in the core. Almost complete angular momentum exchange between the core and mantle for the time-scale is shown by Jault et al. (1988) (see also Jackson et al. (1993) and Bloxham (1998)). t_1^0 and t_3^0 component of the toroidal flow variation, which can be excited by torsional oscillation (Braginsky (1970)), seem responsible to the variatioin of the angular momentum in the core for the time-scale. The variation of the two velocity representing the torsional oscillation is given as the specified velocity field in the problem and magnetic field and other quantities are calaulated.

3 Reference

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