Wavelet analysis of gravity and topography in a spherical geometry

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Abstract. The recent use of multi-dimensional wavelets has shown greatly the ability for capturing distinct features at different scales. In particular, two-dimensional wavelet analysis of non-hydrostatic geoid anomalies has revealed the strong anomalies at plate margins for both oceanic and continental plate boundaries, using a degree 90 geoid. In this study we have extended the previous geoid analysis by wavelets to degree 360 (Le Moine et al., 1996). The convergence of the earlier wavelet solutions is demonstrated clearly by comparing the various wavelet truncation levels up to degree 256. We have also compared these results with a nonlinear band-pass filter in the spectral domain where a Gaussian filter with the logarithm of the degree ℓ acting as the argument has been employed. There are two parameters in this filter, the peak of the logarithm in the degree $log_2(\ell_c)$ and the width w describing the spread in the Gaussian filter in $log_2(\ell)$ space. By using the spatial correlation function between the wavelet geoid solution and the filtered geoid, we are able to estimate the optimal parameters in the width and shift in the peak of $log_2(\ell_c)$ for providing an excellent visual fit between the two types of solutions. This has been conducted for both global and local wavelet solutions, up to degree 256. However, the width of the optimal solution is extremely broad. A wavelet solution, with a length-scale corresponding to degree 256, would need a filter with over 300 spherical harmonics centering around $\ell_c = 157$ for an optimal spatial fit. For length-scales corresponding to degree 128 and 64, the respective filters because of the logarithmic nature of the argument in the Gaussian filter must also have the same 300 spherical harmonics but with ℓ_c centering respectively at 78 and 40. Thus the computational effort with the band-pass filter exceeds greatly those associated with wavelets.

1 Band-pass filter

In order to suppress artificial oscillation signal by truncation, we applied a Gaussian filter to geoid data in wavelength-domain of spherical harmonics, defined as below;

$$P(l) = exp\left(-\left(\frac{log_2(l/l_c)}{w}\right)^2/2\right), \qquad (1)$$

where l is a degree in spherical harmonics, l_c is a center of degree for the Gaussian filter, and w is a width of the filter. The effect of w on the filter can be seen in the Fig. 1.

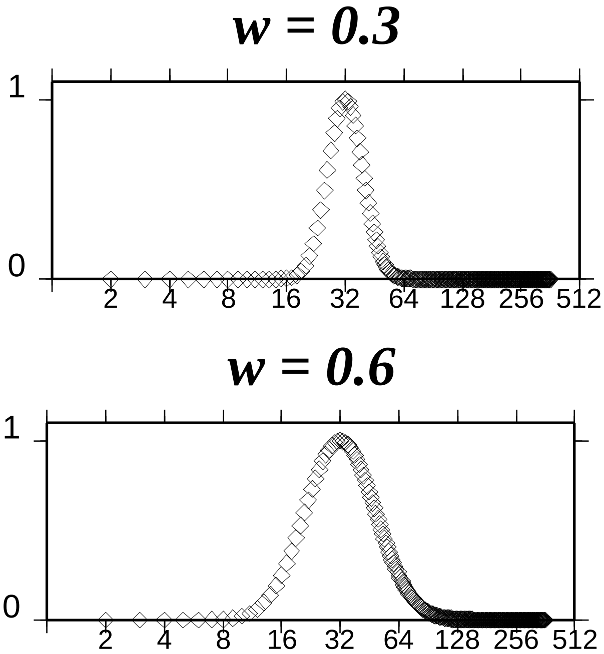

Fig. 1. Gaussian filter used in this study. The filter is symmetrical with a scale of log_2 in degree.

2 Comparison

First we compare geoids between wavelet analysis and band-pass filter in spherical harmonics with corresponding lc. For example, we show the case of wavelength corresponds $l = 32$ in Fig. 2. Fig. 2a, the top figure, is the wavelet analysis for $1/a = 32$. On the other hand, (b) is the band-pass filtered geoid with a Gaussian filter, where $l_c = 32$ and $w = 0.6$ in eq. 1. Apparently, band-pass filter emphasizes shorter wavelength and Gibbs oscillation can be found. Looking at Fig. 2c, narrower bandwidth $w = 0.3$ enhances this feature.

3 Best-fit parameters

Considering the nature found in previous section, we determine the best-fit parameters for l_c and w by maximize cross correlation of geoids between wavelet analysis and band=pass filter in a spatial domain.

For the case $1/a = 32$, we obtained the best-fit parameters that $l_c = 19.7$ and $w = 1.0$. Value of the l_c is expected from the feature in the previous section. In addition we need considerably large value of w to fit the wavelet geoid. We only show the Gaussian filters for $w = 0.3$ and $w = 1.0$ in Fig. 1, however, we can easily imagine the width of filter for $w = 1.0$ since width is proportional to the w. The filter with $w = 1.0$ cover almost all the wavelength in spherical harmonics.

This feature is completely hold other wavelength from $1/a = 2$ and even up to $1/a = 256$.

4 Conclusion

We have compared geoid between wavelet analysis and band-pass filter in the wavelength domain. We found that center of band-pass filter must be shifted toward lower degree to reconstruct wavelet transformed geoid. In addition, it is much more important, the band width of the filter must be quite wide, which almost covers entire degree range.

This kind of quantitative comparison between wavelet analysis and classical band-pass filter will provide formulation of admittance study, such as geoid and topography, in wavelet transforms.

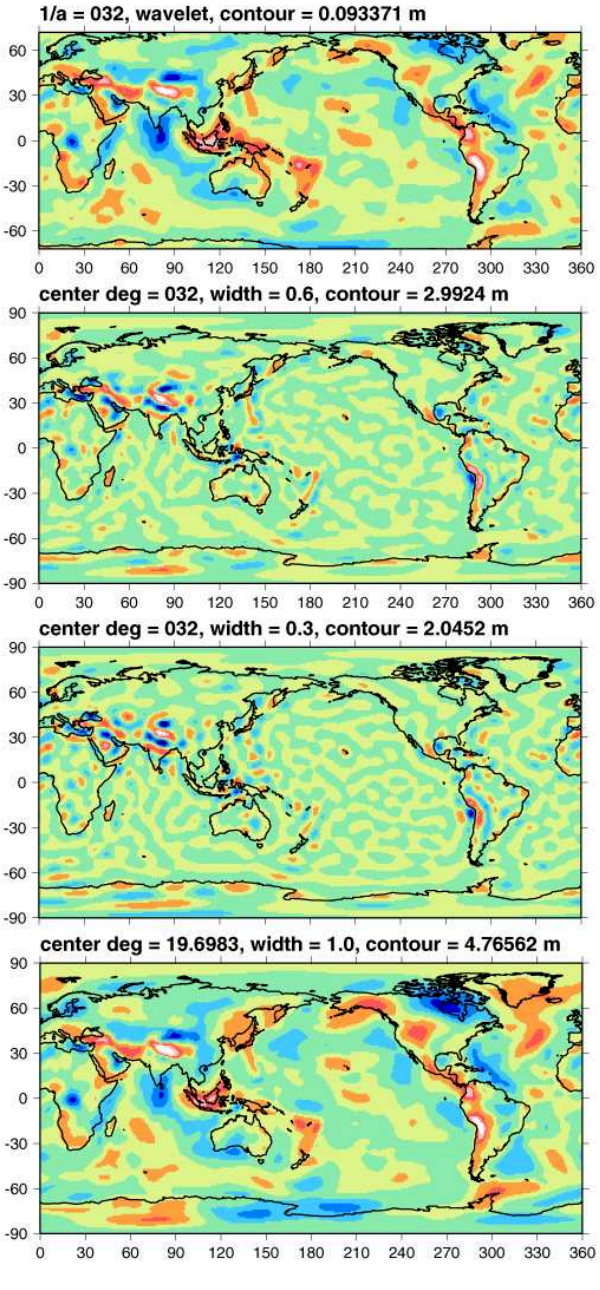

Fig. 2. Geoid comparison between (a) wavelet ($1/a = 32$) analysis, (b) band-pass filter of lc = 32,w=0.6, (c) c = 32,w=0.3, $and(d)best - fitparameterof$lc=19.7m $w = 1.0$. Blue region is geoid low and red region is high.